ITPS AK - 20/04

Linguaggio → Grammatica

Esercizio 1

$$L = \{ a^n \mid n = 2k, \quad k > 0 \}$$

$$= \{ a^2, a^4, \dots \}$$

Il numero delle a è pari, ma non è 0.

$G(L)$: $S \to aa \mid aaS$ Tipo 3

Esercizio 2

$$L = \left\{ a^n b^k c^m \;\middle|\; \begin{array}{l} n > 0 \\ m > 0 \\ k > n+m \end{array} \right\}$$

$$= \{ abbbc, a^2 b^4 c, \dots \}$$

$$= \left\{ a^n \underbrace{b^{n+m+x}} c^m \;\middle|\; \begin{array}{l} n > 0 \\ m > 0 \\ x > 0 \end{array} \right\}$$

$$= \left\{ a^n b^n \underbrace{b^m} b^x c^m \;\middle|\; \begin{array}{l} n > 0 \\ m > 0 \\ x > 0 \end{array} \right\}$$

$$= \left\{ a^n b^n \, b^x \, b^m c^m \;\middle|\; \begin{array}{l} n > 0 \\ m > 0 \\ x > 0 \end{array} \right\}$$

$$L = \underset{L_1}{\overset{n>0}{a^n b^n}} \cdot \underset{L_2}{\overset{x>0}{b^x}} \cdot \underset{L_3}{\overset{m>0}{b^m c^m}}$$

$G(L_1)$: $S_1 \rightarrow a S_1 b \mid ab$ Tipo 2

$G(L_2)$: $S_2 \rightarrow b S_2 \mid b$ Tipo 3

$G(L_3)$: $S_3 \rightarrow b S_3 c \mid bc$ Tipo 2

$G(L_1 \cdot L_2 \cdot L_3)$:

$$\begin{aligned} S &\rightarrow S_1 S_2 S_3 \\ S_1 &\rightarrow a S_1 b \mid ab \\ S_2 &\rightarrow b S_2 \mid b \\ S_3 &\rightarrow b S_3 c \mid bc \end{aligned}$$

$a^3 b^5 c \Leftarrow S$

1. $S \rightarrow S_1 S_2 S_3$ $\qquad S_1 S_2 S_3$
2. $S_1 \rightarrow a S_1 b$ $\qquad a S_1 b S_2 S_3$
3. $S_1 \rightarrow a S_1 b$ $\qquad aa S_1 bb S_2 S_3$
4. $S_1 \rightarrow ab$ $\qquad aaabbb S_2 S_3$
5. $S_2 \rightarrow b$ $\qquad aaabbbb S_3$
6. $S_3 \rightarrow bc$ $\qquad aaabbbbbc$

Esercizio 3

$L = \{ a^i b^k c^j \mid i > 0,\ j > 0,\ 0 \leq k \leq i+j \}$
$= \{ ac,\ abc,\ abbc,\ aac,\ aabc,\ \dots \}$

$$0 \leq K_i + K_j \leq i + j$$

$L_1 = \{ a^i b^{k_i} \mid 0 \leq K_i \leq i,\ i > 0 \} = \{ a,\ ab,\ a^2 b\ \dots \}$

$L_2 = \{ b^{k_j} c^j \mid 0 \leq K_j \leq j,\ j > 0 \} = \{ c,\ bc,\ bc^2\ \dots \}$

$G(L_1)$: $S_1 \to a \mid aS_1 \mid aS_1 b \mid ab$

$G(L_2)$: $S_2 \to c \mid S_2 c \mid bS_2 c \mid bc$

$L_1 \cdot L_2$: $S \to S_1 S_2$
$S_1 \to \dots$
$S_2 \to \dots$

Grammatica $\to$ Linguaggio

Esercizio 4

$P = \{ S \to aSb \mid aBb \mid aCb$

$B \to$ ...

$B \to a \mid aB \mid aaaB$

$C \to \lambda \mid aC \mid bC \}$

$a \ldots a \qquad b \ldots b \qquad S \to aSb \;]$

$\vdots$

$S \to aSb$

$S \to aBb$

$a \ldots a \; B \; b \ldots b$

$S \to aCb$

$a \ldots a \; C \; b \ldots b$

$$L = \{ a^n w b^n \mid w \in \{a, b\}^*, n > 0 \}$$

$\{X\}^*$ comprende λ

$\{X\}^+$ NON comprende λ

$$S \overset{k}{\Rightarrow} a^k S b^k$$

1. $a^k (aBb) \, b^k$

$a^{k+1} \quad B \quad b^{k+1}$

a^n

$\Rightarrow a^{k+1+n} \, b^{k+1}$

(2) $a^k \underline{(aCb)} b^k$

$a^{k+1} \; C \; b^{k+1}$

$\Rightarrow \; a^{k+1} \, w \, b^{k+1}$

$w \in \{a,b\}^*$

Esercizio 5

$P = \{ S \to aSc \mid aBC$

$B \to bB \mid \lambda$

$C \to cc \mid cC \}$

$L = \{ a^n b^m c^J \mid m \geq 0 , \; n > 0 , \; J > n+1 \}$

Automi a pila
Pumping Lemma

$L = \{ wcw^R \quad w \in \{a,b\}^* \}$

$L = \{ w \in \{a,b\}^* \mid \#a = \#b \}$

$L = \{ a^n b^n c^n \mid n \geq 0 \}$

$L = \{$ quadrato perfetto $\}$

$L = \{ a^n \mid n$ primo $\}$

Esercizio 6

$L = \{ a^n b^n \mid n \geq 0 \}$

(a) Pumping Lemma

(1) Sia L un linguaggio regolare. Allora esiste una costante $N > 0$ tale che (2) $\forall z \in L$ $|z| \geq N$ possiamo suddividere z in tre sottostringhe $z = uvw$ tali che

(1) $|uv| \leq N$

(2) $v \neq \lambda$

(3) $\forall i \geq 0,\ uv^iw \in L$ (3)

Applicazione

Step 1. Assumo L regolare

Step 2 Scegliere una $z \in L$, $|z| \geq N$

Step 3. Confutare la (3) per tutte le scomposizioni $z = uvw$

$\exists i \geq 0,\ uv^iw \notin L$

$$z = a^N b^N \qquad |z| = 2N > N \qquad >$$

$$|uv| \leq N \qquad \underbrace{a^{N-1}}_{u}\ \underbrace{a}_{v} \qquad \underbrace{\lambda}_{u}\ \underbrace{a^N}_{v}$$

① $v = a^k \qquad 1 \leq k \leq N$ $\qquad \cancel{\underbrace{a^N}_{u}\ \underbrace{\lambda}_{v}}$

$v \neq \lambda$ per la (2)

$$uv^2w = a^{N+k} b^N$$

Vincolo : $|a| = |b|$

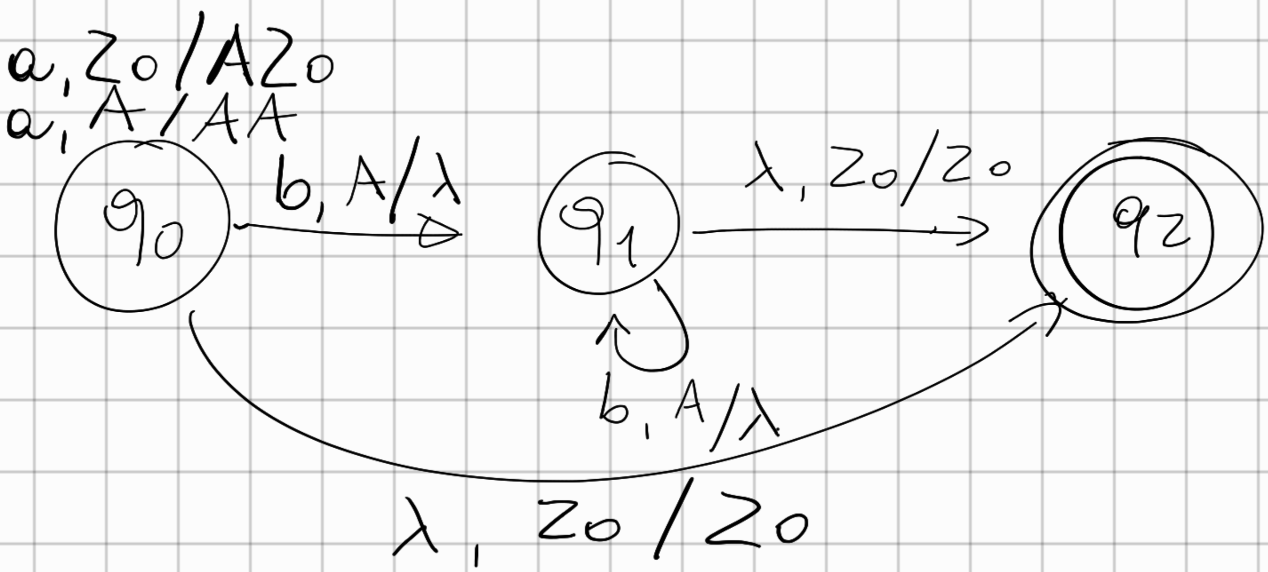
Ma in uv^2w $|a| \neq |b|$, pertanto

$uv^2w \notin L$

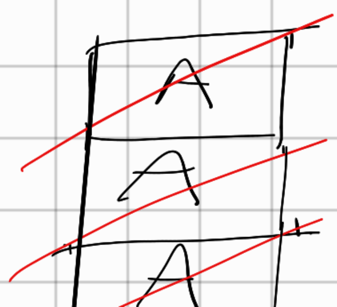
Contraddizione con la (3).
La contraddizione deriva dall'aver assunto L regolare.
Pertanto L non è regolare

- Automa a pila $L = \{a^n b^n \mid n \geq 0\}$

simbolo top della pila	stato	λ	a (Σ)	b
Z_0	q_0	(q_2, Z_0)	(q_0, AZ_0)	—
A	q_0		(q_0, AA)	(q_1, λ)
A	q_1		—	(q_1, λ)
Z_0	q_1	(q_2, Z_0)	—	



$a^3 b^3$



$a^3 b^3 \longrightarrow$ $\boxed{Z_0}$

Esercizio A

$$L = \{ a^i b^j c^k \mid i + j = k \}$$

- PL
- Automa

Esercizio B

$$L = \{ a^n b^{2n} \mid n \geq 0 \}$$

- PL
- Automa

Esercizio C

$$L = \{ a^i b^j \mid 0 \leq i \leq j \}$$

- PL
- Automa