ITPS LZ - 16/04

Esercizio 4 Traccia 2023

$L_1 = \{ w = a^n b^k \;,\; n, k > 0 \;,\; n \geq k \}$

$L_2 = \{ w = a^n b^k \;,\; n, k > 0 \;,\; n \leq k \}$

$L_1 = \{ ab,\ a^2b,\ a^2b^2,\ a^3b,\ \dots \}$

$L_2 = \{ ab,\ ab^2,\ a^2b^2,\ ab^3,\ \dots \}$

$L = L_1 \cap L_2$

L_1: $n > 0$, $k > 0$, $n \geq k$

L_2: $n > 0$, $k > 0$, $n \leq k$

$(n > 0,\ k > 0,\ n \geq k)$ AND $(n > 0,\ k > 0,\ n \leq k)$

(L_1 | L_2)

$n > 0 \quad k > 0 \quad n = k$

$L = \{ w \in X^* \mid w = a^n b^k \;,\; n > 0,\ k > 0,\ n = k \}$

$= \{ w \in X^* \mid w = a^n b^n \;,\; n > 0 \}$

$L = \{ ab, aabb, \dots \}$

$G: S \to ab \mid aSb \qquad L(G)$

G è context-free $\Rightarrow$ L è context-free

Se $L(G) \subseteq L$, allora possiamo concludere

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$$L = \{ w \in X^* \mid w = a^i b^j c^k,\ i < k,\ i > 0,\ k > 0,\ j \geq 0 \}$$

$$= \{ aee, aeee, abcc, abbcc, \dots \}$$

- ogni a viene generata insieme a una c tramite le ultime
- dobbiamo generare almeno una c "da sola"
- le b sono indipendenti dalle a e dalle c

NO

$$= \{ \underbrace{a^n b^m}_{L_1} \cdot \underbrace{c^n c^q}_{L_2} \mid \underset{m \geq 0}{n, q > 0} \}$$

L_1 S_1 L_2 S_2

$G(L_1)$: $S_1 \to aS_1 \mid aB$

$B \to \lambda \mid bB$

$G(L_2)$: $S_2 \to cS_2 \mid c$

non sono collegate!

S1

$L_1 = \{ a^n b^m c^n \mid n > 0,\ m \geq 0 \}$

$L_2 = \{ c^q \mid q > 0 \}$

① ② →

$S \to aSc \mid aBC$

$B \to bB \mid \lambda$

$C \to cc \mid cC$

$S \to AC$

$A \to aAc \mid aBc$

$B \to \lambda \mid bB$

$C \to c \mid cC$

$z = a^2 b^4 c^3$

② 1. $S \to AC$ = AC

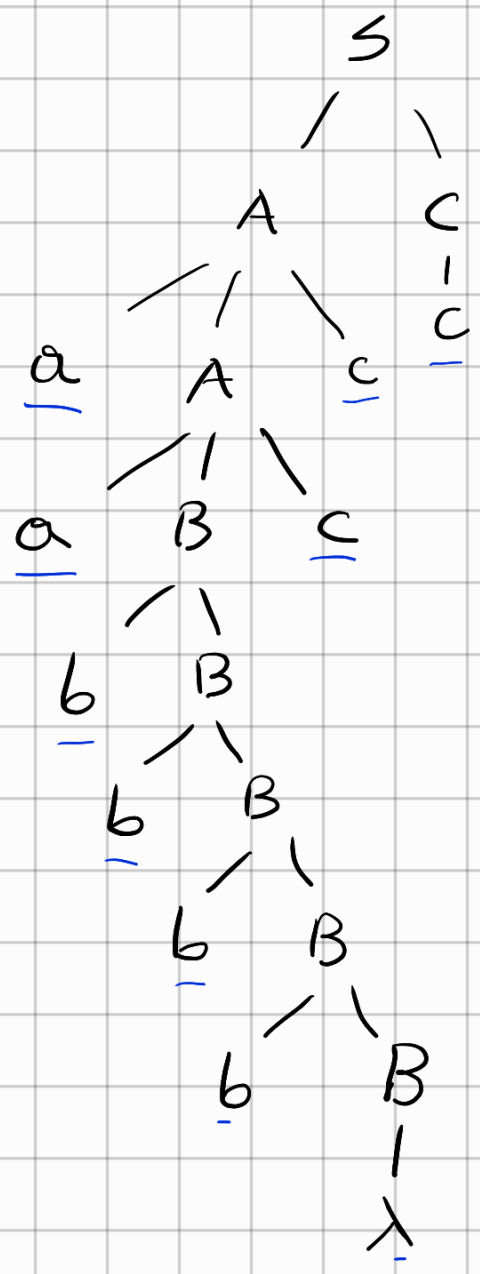
2. $A \to aAc$ = $aAcC$

3. $A \to aBc$ = $aaBccC$

4./7. $B \to bB$ = $aabbbbBccC$

8. $B \to \lambda$ = aabbbbccC

9 $C \to c$ = aabbbbccc ✓



① $z = a^2b^4c^3$

$S \to aSc \mid aBC$

$B \to bB \mid \lambda$

$C \to cc \mid cC$

1. $S \to aSc$ = aSc

2. $S \to aBC$ = aaBCc

3/6. $B \to bB$ = aabbbbBCc

7. $B \to \lambda \quad = \quad aabbbbcc$

8. $C \to cc \quad = \quad aabbbbccc$

Esercizio 2 Traccia 2023

$$P = \{ S \to aSb \mid aBb \mid aCb$$
$$B \to a \mid aB \mid aaaB$$
$$C \to \lambda \mid aC \mid bC \}$$

$L(G)$

$B \to aaaB$

$B \to aB \quad \times 3$

$B \to aB$
$C \to aC$ =

$S \to aCb \quad = \quad aCb$

$C \to \lambda \quad = \quad ab$

$$L(G) = \{ a^n \gamma b^n \mid \gamma \in \{a, b\}^* , \; n > 0 \}$$

$$S \xrightarrow{k} a^k S b^k$$

① $a^k (aBb) b^k$

$a^{k+1} B b^{k+1}$

$$a^n$$

$$\Rightarrow \quad a^{k+1+n} \; b^{k+1}$$

②

$$a^k (a \subset b) \; b^k$$

$$a^{k+1} \subset b^{k+1}$$

$$\Rightarrow \quad a^{k+1} \; \gamma \; b^{k+1}$$

$$\gamma \in \{a, b\}^*$$

$$① \cup ② = \left\{ a^n \gamma \; b^n \;\middle|\; \begin{array}{l} n > 0 \\ \gamma \in \{a, b\}^* \end{array} \right\}$$