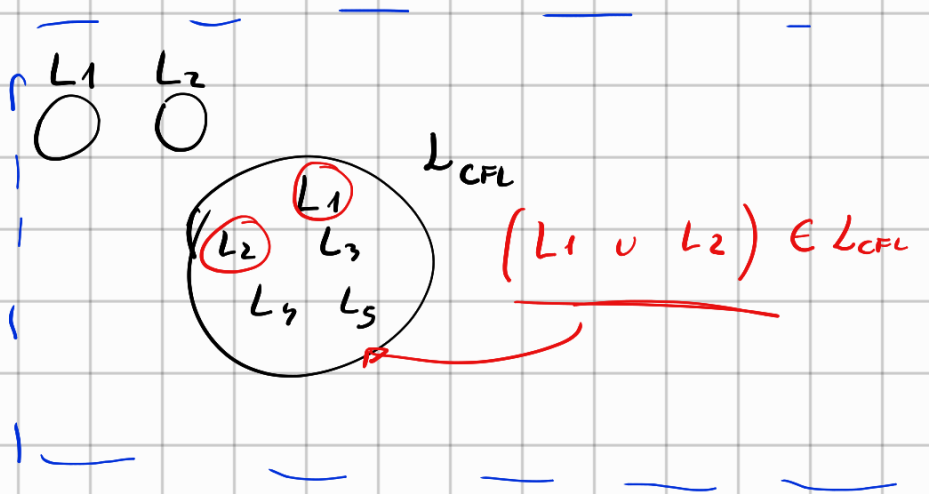


# Unione

①  $L_1 = \{ a^m b^m \mid m > 0 \}$   $X = \{ a, b, c, d \}$   
 $L_2 = \{ c^m d^m \mid m > 0 \}$

$L_1 \cup L_2$  CF  $\rightarrow P_1 : S_1 \rightarrow ab \mid a \underline{S} b$   
 CF  $\rightarrow P_2 : S_2 \rightarrow cd \mid c \underline{S} d$

$G(L_1 \cup L_2) = P_1 \cup P_2 \cup \{ S \rightarrow S_1 \mid S_2 \}$



$L = \{ \underline{a^m b^m} \vee \underline{c^m d^m} \mid m, m > 0 \}$

②  $L_1 = \{ a^m b^k \mid m, k > 0 \}$   
 $L_2 = \{ b^m c^m \mid m > 0 \}$   $X = \{ a, b, c \}$

$G(L_1 \cup L_2) = ?$

$$P_1 \quad L_1 = \{ ab, abb, aab, \dots, a^4 b^5, \dots \}$$

$$\rightarrow S_1 \rightarrow aS_1 \mid aB$$

Typo 3

$$B \rightarrow bB \mid b$$

$$P_2 \quad S_2 \rightarrow bS_2c \mid bc$$

Typo 2

$$\begin{array}{cc} L_1 \cup L_2 & \\ G(L_1) & G(L_2) \\ T_3 & T_2 \end{array}$$

$\downarrow$   
 $T_2$

$$P: P_1 \cup P_2 \cup \{ S \rightarrow s_1, s \rightarrow s_2 \}$$

$\downarrow$   
 $T_2$

$$\textcircled{3} \quad P_1 = \rightarrow S_1 \rightarrow aA \quad \cdot \quad L_1 = \{ a^m b \mid m > 0 \}$$

$$A \rightarrow aA \mid b \quad \cdot \quad \text{Typo 3}$$

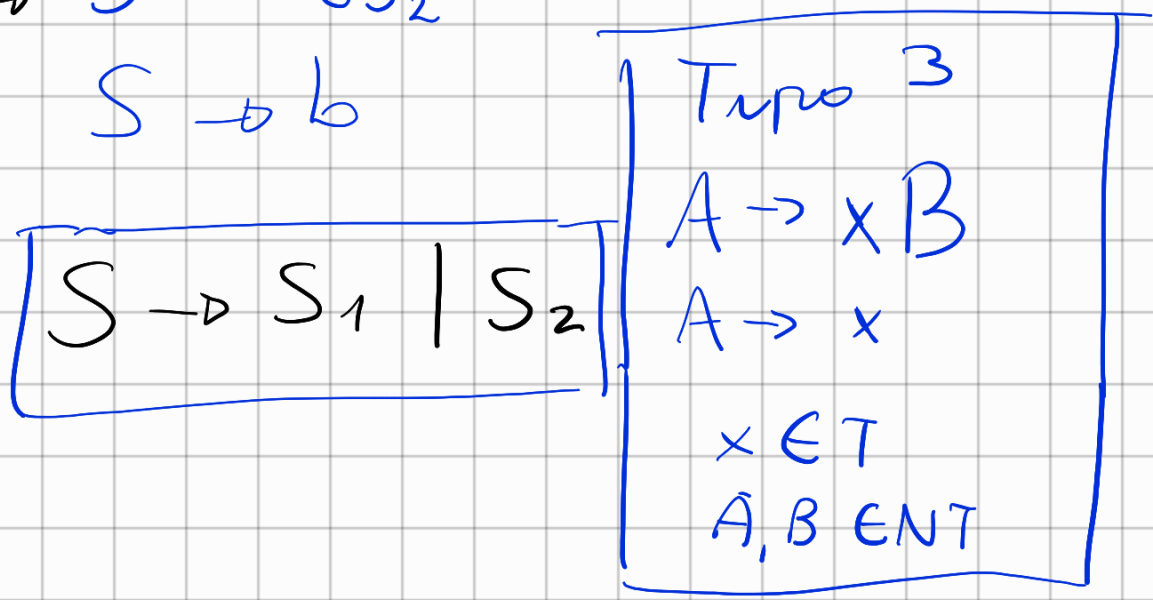
$$P_2 = \rightarrow S_2 \rightarrow bS_2 \mid b \quad \cdot \quad \text{Typo 3}$$

$$L_1 \cup L_2 \quad \begin{array}{l} S_2 \rightarrow (bS_2)^n \\ S_2 \rightarrow (b)^n \end{array} \quad \hookrightarrow L_2 = \{ b^m \mid m > 0 \}$$

$$P: \left\{ S \rightarrow w \mid \begin{array}{l} S_1 \rightarrow w \\ S_1 \rightarrow w \end{array} \in P_1 \right\}$$

$$\left\{ S \rightarrow w \mid S_2 \rightarrow w \in P_2 \right\} \cup P_1 \cup P_2$$

$S \rightarrow aA$   
 $S \rightarrow bS_2$   
 $S \rightarrow b$



Union

$L_2 : \{ S \rightarrow S_1 \mid S_2 \} \cup P_1 \cup P_2$

$L_3 : \{ S \rightarrow w \mid \forall S_1 \rightarrow w \in P_1 \} \cup$   
 $\{ S \rightarrow w \mid \forall S_2 \rightarrow w \in P_2 \} \cup$   
 $P_1 \cup P_2$



concatenation  $\{ aab, aaaab, \dots \}$

①  $L = \{ a^{m+k} b^m \mid m > 0, k > 0 \}$

~~$\{ a^m b^m \cdot a^k \mid m > 0, k > 0 \}$~~

↓

$\{ \underline{aab} \quad aba, oobb \dots \}$

$$= \{ a^k \cdot a^m b^m \mid m > 0, k > 0 \}$$

$$L_1 = \{ a^k \mid k > 0 \}$$

$$L_2 = \{ a^m b^m \mid m > 0 \}$$

$$P_1: S_1 \rightarrow a \mid aS_1 \leftarrow \text{Type 3}$$

$$P_2: S_2 \rightarrow ab \mid aS_2 b \leftarrow \text{Type 2}$$

$$\begin{array}{c} L_1 \cdot L_2 \\ T_3 \quad T_2 \\ \downarrow \\ \underline{T_2} \end{array}$$

$$G(L_1 \cdot L_2): S \rightarrow S_1 S_2 \quad \vee \quad P_1 \quad \vee \quad P_2 \quad |$$

$$a^{(m)} b^{(m)} c^{(m)}$$

$$L_1 \quad L_2 \\ a^m b^m \cdot c^m ?$$

$$S_1 \rightarrow ob \mid aS_1 b \quad S_2 \rightarrow c \mid cS_2$$

$$S \rightarrow S_1 S_2$$

$$L_1 = \{ a^n b, n \geq 0 \}$$

②  $P_1 = \{ S_1 \rightarrow aA, A \rightarrow aA \mid b \}$   
 $P_2 = \{ S_2 \rightarrow bS_2 \mid b \} \cup \{ b^n, n \geq 0 \}$

$$\{ A \rightarrow bB \mid A \rightarrow bB \in P_1 \} ?$$

$$\rightarrow \{ A \rightarrow bS_2 \mid \underline{A \rightarrow b} \in P_1, b \neq \lambda \} ?$$

$$\left. \begin{aligned} & \{ A \rightarrow bS_2 \mid A \rightarrow bB \in P_1, \underline{B \rightarrow \lambda} \in P_1 \} ? \\ & \{ S_1 \rightarrow w \mid S_2 \rightarrow w \in P_2, \underline{S_1 \rightarrow \lambda} \in P_1 \} ? \end{aligned} \right\} \cup P_2$$

$$S_1 \rightarrow aA \quad A \rightarrow aA$$

$$A \rightarrow bS_2$$

$$S_2 \rightarrow bS_2 \mid b$$

③  $G_1 : \{ S_1 \rightarrow aA \mid \lambda, A \rightarrow b \mid bA \}$

$$U_2 = \{ S_2 \rightarrow \underline{e} \mid \underline{e} S_2 \}$$

$$S_1 \rightarrow aA \quad A \rightarrow bA$$

~~$$S_1 \rightarrow \lambda \quad A \rightarrow b$$~~

$$S_1 \rightarrow c \quad A \rightarrow bS_2$$

$$S_1 \rightarrow cS_2 \quad S_2 \rightarrow c \quad S_2 \rightarrow cS_2$$

$$S_1 \rightarrow aA \mid cS_2 \mid c \quad c, abc, abbc, c \dots c$$

$$A \rightarrow bA \mid bS_2$$

$$S_2 \rightarrow c \mid cS_2$$

$$\left\{ \begin{array}{l} k=1 \text{ se } m \neq 0 \\ k=0 \text{ altrimenti} \end{array} \right\}$$

$$L = \left\{ a^k b^m e^m \mid \begin{array}{l} m \geq 0 \\ m > 0 \end{array} \right\}$$