Informatica - 17/04

Esercizio 1

$$L = \{ w \in X^* \mid w = a^k b^n, \; n = k + k^3, \; k > 0 \}$$

$$a^k b^{k+k^3}$$

Stabilire se L è libero da contesto.

Assumiamo L libero da contesto, allora vale il Pumping Lemma.

<ENUNCIATO PL>

$z = a^p b^{p+p^3}$ $\quad |z| = p + p + p^3 > p$ ✓

$z = u\underbrace{vwx}y$

Sottocasi

$|vwx| \leq p$

1. $vwx = a^k$ $\quad 1 \leq k \leq p$
2. $vwx = b^k$ $\quad 1 \leq k \leq p$
3. $vwx = a^t b^s$ $\quad 1 \leq t+s \leq p$

1. $vwx = a^k$ $\quad 1 \leq k \leq p$

$$uv^2wx^2y = a^{p+k'} b^{p+p^3} \quad 1 \leq k' \leq k \leq p$$

$$p+1 \leq \#a \leq p+p$$

$$\underbrace{p+1 \le \#a}\le 2p$$

$$\#b = p + p^3$$

Vincolo : $\#b = \#a + (\#a)^3$

$$\#a + (\#a)^3 \ge p+1 + (p+1)^3 \qquad \#a \ge p+1$$

$$= p+1 + p^3 + 3p^2 + 3p + 1$$

$$= p^3 + 3p^2 + 4p + 2$$

$$\#b = p^3 + p$$

$$p^3 + p \overset{?}{\cancel{=}} p^3 + 3p^2 + 4p + 2$$

$$<$$

$$\Rightarrow \quad uv^2wx^2y \notin L$$

2. $vwx = b^k \qquad 1 \le k \le p$

$$uv^2wx^2y = a^p\, b^{p+p^3+k'} \qquad 1 \le k' \le k \le p$$

$$\#a = p$$

$$p+p^3+1 \le \#b \le p+p^3+p$$

$$p^3+p+1 \le \#b \le p^3 + 2p$$

Vincolo : $\#b = \#a + (\#a)^3$

$$\#b \geq \underbrace{p^3}+\underbrace{p}+\underbrace{1} \; \cancel{\leq}\; > \; \underbrace{p}+\underbrace{p^3}$$

$$\Rightarrow \quad uv^2wx^2y \notin L$$

3. $vwx = a^t b^s \qquad 1 \leq t+s \leq p$

$vx \neq \lambda$

3.1 $\quad v \neq \lambda \quad x = \lambda \quad vwx = a^{t'}$ (1.)

3.2 $\quad v = \lambda \quad x \neq \lambda \quad vwx = b^{s'}$ (2.)

3.3 $\quad v \neq \lambda \quad x \neq \lambda \quad vwx = a^{t'} b^{s'}$

$\hookrightarrow \quad uv^2wx^2y = a^{p+t'} b^{p+p^3+s'} \qquad 1 \leq t'+s' \leq t+s \leq p$

$t' > 0$

$s' > 0$

$$p+1 \leq \#a \leq p+(p-1)$$

$$p+p^3+1 \leq \#b \leq p+p^3+p-1$$

Vincolo: $\#b = \#a + (\#a)^3$

$$\#a + (\#a)^3 \geq p+1+(p+1)^3$$
$$= p+1+p^3+3p^2+3p+1$$
$$= p^3+3p^2+4p+2$$

$$\#b \geq p+p^3+1$$

$$\#b \leq p^3+2 \ldots$$

$\# b = p^3 + 2p - 1$

$$p^3 + 2p - 1 \;\cancel{\overset{?}{=}}\; p^3 + 3p^2 + 4p + 2$$

<

$$\# b \qquad\qquad \# o + (\# o)^3$$

— — — —

$$\# b \geq p^3 + p + 1$$

$$\# o \leq 2p - 1$$

$$\# o + (\# o)^3 \leq 2p - 1 + (2p-1)^3$$

Stesso conclusione

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$\Rightarrow uv^2wx^2y \notin L$

In tutti i sottocasi, $uv^2wx^2y \notin L$

$\Rightarrow$ Contraddizione $\Rightarrow$ L non è libero da contesto

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Esercizio 2

$$L = \{ a^k b^i \mid i = k^3 - k,\; k > 1 \}$$

Stabilire se L è libero da contesto

Suppongo L libero da contesto.

Allora vale il Pumping Lemma.

<ENUNCIATO PL>

$z = a^{p+2} b^{(p+2)^3-(p+2)}$ $\qquad |z| = p+2+(p+2)^3-(p+2) > p$

Sottocasi

1. $vwx = a^k$ $\qquad 1 \leq k \leq p$

2. $vwx = b^k$ $\qquad 1 \leq k \leq p$

3. $vwx = a^t b^s$ $\qquad 1 \leq t+s \leq p$

1. $vwx = a^k$ $\qquad 1 \leq k \leq p$

$$uv^2wx^2y = a^{p+2+k'} b^{(p+2)^3-(p+2)} \qquad 1 \leq k' \leq k \leq p$$

$$p+2+1 \leq \#a \leq p+2+p$$

$$\#b = (p+2)^3-(p+2)$$

Vincolo: $\#b = (\#a)^3 - \#a$

$$\#a \geq p+3$$

$$(\#a)^3-(\#a) \geq (p+3)^3-(p+3)$$

$$(p+2)^3-(p+2) \neq (p+3)^3-(p+3)$$

$\Rightarrow uv^2wx^2y \notin$

$\Rightarrow uv\ w\ x\ y \notin L$

2. $vwx = b^k \qquad 1 \leq k \leq p$

$$uv^2wx^2y = a^{p+2}\, b^{(p+2)^3-(p+2)+k'}$$

$$1 \leq k' \leq k \leq p$$

$$\#a = p+2$$

$$(p+2)^3-(p+2)+1 \leq \#b \leq (p+2)^3-(p+2)+p$$

$$(p+2)^3-(p+2)+1 \neq (p+2)^3-(p+2)$$

$$(\#a)^3 - \#a$$

$\Rightarrow uv^2wx^2y \notin L$

3. $vwx = a^t b^s$

(1) = 3.1 $\quad v \neq \lambda \ , \ x = \lambda \qquad vwx = a^{t'}$

(2) = 3.2 $\quad v = \lambda \ , \ x \neq \lambda \qquad vwx = b^{s'}$

3.3 $\quad v \neq \lambda \ , \ x \neq \lambda \qquad vwx = a^{t'} b^{s'}$

$$uv^2wx^2y = a^{p+2+t'}\, b^{(p+2)^3-(p+2)+s'}$$

$$p+2+1 \leq \#a \leq p+2+p-1$$

$$(p+2)^3-(p+2)+1 \leq \#b \leq (p+2)^3-(p+2)+p-1$$

max #b

min $(\#a)^3 - \#a$

$$(p+2)^3 - (p+2) + p - 1 \stackrel{?}{=} (p+3)^3 - (p+3)$$

$$(p+2)^3 - \cancel{p} - 2 + \cancel{p} - 1$$

$$(p+2)^3 - 3 \neq (p+3)^3 - (p+3)$$

$$\Rightarrow uv^2wx^2y \notin L$$

L non è libero da contesto.

Esercizio 3

$$L = \{ a^i b^j c^k \mid j = \min(i,k) \quad i,k > 0 \}$$

$z = a^p b^p c^p$

1. $vwx = a^k \rightarrow$
2. $vwx = b^k$
3. $vwx = c^k \rightarrow$
4. $vwx = a^t b^s$
5. $vwx = b^t c^s$

uv^2wx^2y

$p+1$	$\#a$
p	$\#b$
p	$\#c$

$\Rightarrow \in L$

uv^0wx^0y

$p-1$	$\#a$
p	$\#b$
p	$\#c$

$\Rightarrow \notin L$

$\hookrightarrow$ equivalente a 4.

$uv^2wx^2y \notin L$

$uv^0wx^0y \notin L$ equivalente

$\Rightarrow uv^2wx^2y = a^{p+t'}b^{p+s'}c^p$

$\#b \neq \min(\#a, \#c)$

$\Rightarrow uv^2wx^2y \notin L$

Extra

1 $L_1 = \{ w = a^n b^k \quad n, k > 0 \quad n \geq k \}$

$L_2 = \{ w = a^n b^k \quad n, k > 0 \quad n \leq k \}$

Trovare $L = L_1 \cap L_2$

2 $P = \{ S \to aSb \mid aBb \mid aCb$

$B \to a \mid aB \mid aaaB$

$C \to \lambda \mid aC \mid bC \}$

Definire L generata da P

3 $L = \{ w \in X^* \mid w = a^i b^j c^k \quad i < k$

$i > 0$

$k > 0$

$j \geq 0$?

- Trovare una grammatica
- Albero di derivazione $z = a^2 b^4 c^3$