

Esercizio 1

$$L_1 = \{ a^m b^m \mid m > 0 \}$$

$$L_2 = \{ c^m d^m \mid m > 0 \}$$

1/2. classe di appartenenza / promittente

$$G(L_1): S_1 \rightarrow a S_1 b \mid ab$$

$$G(L_2): S_2 \rightarrow c S_2 d \mid cd$$

$$G(L_1) \rightarrow \text{Tipo 2}$$

$$G(L_2) \rightarrow \text{Tipo 2}$$

3. Tipo $G(L_1 \cup L_2) \rightarrow$ Tipo 2

4. $G(L_1 \cup L_2):$

$$G(L_1 \cup L_2)$$

$$P_1 \quad \boxed{L_1 \quad S_1} \quad S \rightarrow S_1$$

$$P_2 \quad \boxed{L_2 \quad S_2} \quad S \rightarrow S_2$$

$$P = \{ S \rightarrow S_1 \mid S_1 \rightarrow \dots \in P, \dots \}$$

$$\{S \rightarrow S_2 \mid S_2 \rightarrow w \in P_2\} \cup \underbrace{P_1} \cup \underbrace{P_2}$$

$$G(L_1 \cup L_2) : \begin{aligned} S &\rightarrow S_1 \mid S_2 \\ S_1 &\rightarrow aS_1b \mid ab \\ S_2 &\rightarrow cS_2d \mid cd \end{aligned}$$

$$L_1 \cup L_2 = \{ab, ed, aabb, ccdd, \dots\}$$

5. Descrizione informale di $L_1 \cup L_2$

$L_1 \cup L_2$ contiene tutte le stringhe formate dallo stesso numero di a seguite dallo stesso numero di b, oppure dallo stesso numero di c seguite dallo stesso numero di d.

Esercizio 2

$$L_1 = \{a^m b^k \mid m, k > 0\}$$

$$L_2 = \{b^u c^u \mid u > 0\}$$

(a) classe L_1, L_2

(b) $G(L_1), G(L_2)$

(e) $G(L_1 \cup L_2)$

(d) classe $L_1 \cup L_2$

(a) $P_1: S_1 \rightarrow AB$ $S_1 \rightarrow aS_1 \mid aB$
 $A \rightarrow a \mid aA$ ^{oppure} $B \rightarrow b \mid bB$
 $B \rightarrow b \mid bB$

$$a^m b^k = a^m \cdot b^k$$

$G(L_{11}) G(L_{12})$

$P_2: S_2 \rightarrow bS_2c \mid bc$

$L_1 \rightarrow$ Tipo 3 (lineare destra)

$L_2 \rightarrow$ Tipo 2 (context-free)

(c) $G(L_1 \cup L_2): S \rightarrow S_1 \mid S_2$
 $S_1 \rightarrow aS_1 \mid aB$
 $B \rightarrow bB \mid b$
 $S_2 \rightarrow bc \mid bS_2c$

(d) tipo $L_1 \cup L_2$

$G(L_1 \cup L_2) \rightarrow$ tipo 2

Tipo 3 \cup Tipo 2 \Rightarrow Tipo 2

Esercizio 3

$$L_1 = \{ a^m b \mid m \geq 0 \} \quad P_1: S_1 \rightarrow a S_1 \mid b$$

$$L_2 = \{ b^m \mid m > 0 \} \quad P_2: S_2 \rightarrow b \mid b S_2$$

$G(L_1) \rightarrow$ Tipo 3

$G(L_2) \rightarrow$ Tipo 3

$$G(L_1 \cup L_2): \quad S \rightarrow S_1 \mid S_2 \\ S_1 \rightarrow a S_1 \mid b \\ S_2 \rightarrow b \mid b S_2$$

Esercizio 4

$$L_1 = \{ a^m b^n \mid m > n \geq 0 \}$$

$$L_2 = \{ a^m b^n \mid 0 \leq m < n \}$$

$$L_1 = \{ a, aab, aa, aaab, \dots \}$$

$$L_2 = \{ b, bb, abb, abbb, \dots \}$$

$aabb \in L_1$? No

$aa^m b^m \in L_2$? No

$$L_1 \cup L_2 = \left\{ a^m b^m \mid m \neq n, \begin{matrix} m \geq 0 \\ n \geq 0 \end{matrix} \right\}$$

$$G(L_1): S_1 \rightarrow aS_1b \mid aS_1 \mid a$$

$$G(L_2): S_2 \rightarrow aS_2b \mid bS_2 \mid b$$

$$G(L_1 \cup L_2): S \rightarrow S_1 \mid S_2$$

$$S_1 \rightarrow \epsilon S_1 b \mid a S_1 \mid a$$

$$S_2 \rightarrow \epsilon S_2 b \mid b S_2 \mid b$$

$$\overline{L_1 \cup L_2} = \left\{ a^m b^m \mid m = n, \begin{matrix} m \geq 0 \\ n \geq 0 \end{matrix} \right\}$$

$$= \left\{ a^m b^m \mid m \geq 0 \right\}$$

Esercizio 5

$$L = \left\{ a^{m+k} b^m \mid m > 0, k > 0 \right\}$$

(a) Riscrivere L come $L_1 \cdot L_2$

$$L_2 = \left\{ a^m b^m \mid m > 0 \right\}$$

$$L_1 = \{a^k \mid k > 0\}$$

$$a^m b^m a^k \neq a^k a^m b^m$$

$$P_1: S_1 \rightarrow aS_1 \mid a \quad \text{Tipo 3}$$

$$P_2: S_2 \rightarrow aS_2 b \mid ab \quad \text{Tipo 2}$$

$$G(L_1 \cdot L_2): S \rightarrow S_1 S_2 \quad \cup P_1 \cup P_2$$

$$G(L_1 \cdot L_2): \begin{array}{l} S \rightarrow S_1 S_2 \\ S_1 \rightarrow a \mid aS_1 \\ S_2 \rightarrow ab \mid aS_2 b \end{array} \quad \text{Tipo 2}$$

Esercizio 6

$$\{a^k b^m \mid k \geq 0, m \geq 0\}$$

$$G_1: S_1 \rightarrow aA \mid \lambda \quad A \rightarrow aA \mid b \quad \text{Tipo 3}$$

$$G_2: S_2 \rightarrow e \mid cS_2 \quad \text{Tipo 3} \quad \{c^u \mid u > 0\}$$

$$S_1 \rightarrow aA \mid c \mid cS_2$$

$$A \rightarrow aA \mid bS_2$$

$$a^k b^m c^n \quad \begin{array}{l} k \geq 0 \\ m \geq 0 \\ n > 0 \end{array}$$

$$\Sigma_2 \rightarrow C \mid C \Sigma_2$$

Esercizio 7

$$L_1 = \{ w = a^m b^k \mid m, k > 0, m \geq k \}$$

$$L_2 = \{ w = a^m b^k \mid m, k > 0, m \leq k \}$$

$$L_1 \cap L_2$$

$$L_1 = \{ ab, a^2b, a^2b^2, a^3b, a^3b^3, \dots \}$$

$$L_2 = \{ ab, ab^2, a^2b^2, ab^3, a^3b^3, \dots \}$$

$$L_1 \cap L_2 =$$

$$m, k > 0$$

$$m \geq k$$

AND

$$m, k > 0$$

$$m \leq k$$

$$= m, k > 0 \quad m \geq k \text{ AND } m \leq k$$

$$\Rightarrow m = k$$

$$= \{ a^m b^k \mid m, k > 0, m = k \}$$

$$= \{ a^m b^m \mid m > 0 \}$$

$$= \{ ab, a^2 b^2, \dots \}$$

Esercizio A

$$L = \left\{ w = a^i b^j c^k \mid \begin{array}{l} i < k, \quad i > 0 \\ \quad \quad \quad k > 0 \\ \quad \quad \quad j \geq 0 \end{array} \right\}$$

- Decomporre L in $L_1 \cdot L_2$
- Trovare la grammatica
- Determinare la classe di L

Esercizio B

$$L_1 = \{ a^i b^j c^k \mid i = j, \quad i, j, k > 0 \}$$

$$L_2 = \{ a^i b^j c^k \mid j = k, \quad i, j, k > 0 \}$$

- Trovare $G(L_1), G(L_2)$
- Determinare $L_1 \cap L_2$
- Definire la classe di $L_1 \cap L_2$